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GRADIENT CYBERNETICS

The Calculus of Recursion

SYNTHESIS ESSAY III

The Geometric Field

The Derivation of the Registration Sphere $S^2(0.6)$

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Gradientology

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ABSTRACT

Essays I and II established the complete informational and mechanical ground of the Gradientology: the primitive suite {E = 0.8, C = 0.7, F = 0.6, δ = 0.1}, the Hardlock constants, and the informational necessity of the discrete lattice. One gap remained open: G-2, the geometric architecture of Registration. The derivation asserted that the Registration primitive F defines the boundary of a three-dimensional configuration space, but the derivation of the spherical topology of that boundary from the isotropy of the Phase I field was deferred. This essay closes G-2 in full. We begin from the mutual functional independence of the three primitives, which necessitates three orthogonal basis vectors spanning a three-dimensional configuration space C^3. The Phase I ground state G = E × C × F is a scalar product of three orthogonal quantities: it is rotationally symmetric in C^3. We prove that rotational symmetry forces the unique closed boundary of the relational field to be a sphere. Every alternative closed surface — cube, torus, ellipsoid — introduces directional bias inconsistent with the isotropy of the Phase I field. The sphere S^2 is the only closed surface in which every surface point is equidistant from the origin with no preferred orientation. The radius of this sphere is derived as r = F = 0.6: F is the Registration primitive, which defines the boundary at which the relational content of the generative dyad (E × C) becomes commensurable with the reference medium. Below F, signal cannot be distinguished from noise; F is the Surface of Fact-Making. In Phase I, F is suppressed inside the multiplicative product and the sphere exists as latent architecture — defining the field's boundary without performing registration. The Inversion Principle liberates F to the denominator, activating the sphere's surface as the registration manifold. The Phase II kinetic helicon — the worldline generated by E and C propagating along the F-axis — inscribes an elliptical sweep on the sphere's surface with cross-sectional area $A_{EC} = E \times C = 0.56$. The solid angle subtended by this ellipse on the sphere is: $\Omega = A_{EC} / r^2 = EC/F^2 = 0.56/0.36 = 14/9 \approx 1.556$ sr. This is the fractional activation of F's registration surface per chronon. We derive the dimensional progression $[T^{-3}] \rightarrow [T^{-1}] \rightarrow$ dimensionless as the geometric counterpart of the algebraic phase transitions. We prove that d = 3 is uniquely required: d < 3 cannot clear the resolution remainder $\varepsilon_{snap} = 1/30$; d > 3 is a thermodynamic sink. G-2 is closed.

Keywords: G-2 · Registration Sphere · $S^2(0.6)$ · Configuration Space C^3 · isotropy · rotational symmetry · Surface of Fact-Making · kinetic helicon · solid angle · $\Omega = 1.556$ sr · EC/F^2 · d = 3 uniqueness · Phase I latency · Inversion Principle · dimensional progression · resolution remainder · inversive clearance · Hardlock

Derivational Profile

Table 0. Essay Derivational Profile

Attribute	Content
Essay Number	III
Series	Gradient Cybernetics: The Calculus of Recursion
Title	The Geometric Field — The Derivation of the Registration Sphere $S^2(0.6)$
Primary Gap(s)	G-2 — Spherical geometry of F; $S^2(0.6)$ from isotropy; $\Omega = EC/F^2 = 1.556$ sr
Input Constants	$E = 4/5 = 0.8$, $C = 7/10 = 0.7$, $F = 3/5 = 0.6$, $\delta = 1/10 = 0.1$ [all inherited from Hardlock]
Core Derivation	C^3 from orthogonality; $S^2(0.6)$ from isotropy; $r = F = 0.6$; $\Omega = EC/F^2 = 14/9 \approx 1.556$ sr
Key Result	G-2 closed. Registration Sphere $S^2(0.6)$ derived. Solid angle $\Omega = 1.556$ sr confirmed. Phase III architecture grounded.
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1. Introduction

Essays I and II closed the foundational derivational chain. Essay I established the Hardlock: the complete zero-parameter derivation from absolute nothing to the primitive suite, the Phase I/II transition, and all mechanical constants. Essay II closed gap G-1: the informational ground, rigorously deriving I_min, $\delta = 0.1$, $F = 0.6$ as information-theoretic attractor, $\rho = 0.04$, and N_sat = 25. One gap remained open in the Hardlock register: G-2, the geometry of Registration.

G-2 asks: why is the configuration space of the ***Veldt*** a three-dimensional sphere, and why does that sphere have radius exactly $F = 0.6$? The Hardlock asserted F as the Registration floor and the field as a three-dimensional manifold, but the derivation of the specific spherical topology from the isotropy of the Phase I field was not carried to completion. This essay carries it to completion.

The derivation proceeds in four stages. Stage one derives the three-dimensional configuration space C^3 from the mutual functional independence and orthogonality of the three primitives. Stage two proves that the Phase I ground state is rotationally symmetric in C^3, and that this symmetry uniquely forces the boundary of the relational field to be a sphere. Stage three derives the radius $r = F = 0.6$ as the registration boundary — the Surface of Fact-Making — and establishes the latency of F in Phase I. Stage four derives the Phase II kinetic helicon inscribed on the sphere's surface and computes the solid angle $\Omega = EC/F^2 = 14/9 \approx 1.556$ sr.

The result is the complete geometric architecture of the ***Veldt***: C^3 as configuration space, $S^2(0.6)$ as the registration boundary, the helicon as the kinetic trajectory, and $\Omega = 1.556$ sr as the fractional surface activation per chronon. Every value is a zero-parameter consequence of the primitive suite. G-2 is closed.

2. The Configuration Space C^3 — From Functional Independence to Dimensionality

Before any geometry can be assigned to the ***Veldt***, the dimensionality of its configuration space must be derived from the primitives themselves. This is not a choice of representation; it is a structural consequence of the functional roles of E, C, and F.

2.1 Mutual Functional Independence of the Three Primitives

The three primitives occupy distinct and non-reducible functional positions in the triadic architecture. Systematisation (E) is the generative drive: the source of potential whose role is to initiate. Constraint (C) is the limitative boundary: the structural threshold whose role is to negate unbounded potential into specific morphospace. Registration (F) is the informational grain: the reference medium whose role is to validate states against noise.

These three roles are mutually non-substitutable. No two primitives can be fused without destroying the third function. E cannot serve as its own boundary (that would collapse drive into constraint, producing a Singularity). C cannot measure the states it limits (that would conflate boundary and registration). F cannot generate potential (that would violate the logical chronology proven in the Triadic Mandate: potential precedes limit precedes measure).

PRINCIPLE — P-2.1 — Functional Independence of the Primitive Triad

The three primitives E, C, F occupy mutually non-substitutable functional positions. No functional collapse between any two primitives is possible without destroying Mediational Closure. This non-substitutability is the algebraic expression of mutual functional independence: each primitive defines the field's state along a dimension that neither of the other two can access. Three independent functional dimensions are present in the triadic structure. No more, no fewer.

2.2 Orthogonality from Functional Independence

Mutual functional independence has a precise geometric consequence. In the configuration space of the ***Veldt***, the state vector of any locus is a function of all three primitive values simultaneously: $v = [\hat{E}, \hat{C}, \hat{F}]$. For this representation to be non-degenerate — for each primitive to contribute an independent degree of freedom to the state vector — the basis vectors corresponding to E, C, and F must be orthogonal.

If any two basis vectors were non-orthogonal, the corresponding primitives would share a component of their variation: part of the change in one would be expressible as a change in another. But functional independence precludes this. The dot products vanish:

DERIVATION — D-2.1 — Derivation of Orthogonality of the Primitive Basis

Let $\hat{e}, \hat{c}, \hat{f}$ be the unit basis vectors corresponding to E, C, F respectively.

Claim: $\hat{e} \cdot \hat{c} = 0$. Proof: If $\hat{e} \cdot \hat{c} \neq 0$, then a unit change in $\hat{e}$ has a non-zero projection onto $\hat{c}$. This means part of the variation in the generative drive is expressible as a variation in the limitative boundary. But E is the source of potential and C is the negation of potential; they are functionally opposite roles with no shared component. If $\hat{e} \cdot \hat{c} \neq 0$, then either E can serve as a partial constraint or C can serve as a partial source — both are violations of the Triadic Mandate. Therefore $\hat{e} \cdot \hat{c} = 0$.

Claim: $\hat{c} \cdot \hat{f} = 0$. Proof: If $\hat{c} \cdot \hat{f} \neq 0$, part of constraint variation is expressible as registration variation. But C demarcates morphospace and F validates states within it; they operate on categorically distinct objects. If $\hat{c} \cdot \hat{f} \neq 0$, F would partially define boundaries — violating the logical chronology. Therefore $\hat{c} \cdot \hat{f} = 0$.

Claim: $\hat{e} \cdot \hat{f} = 0$. Proof: If $\hat{e} \cdot \hat{f} \neq 0$, generative drive partially determines registration grain. But F is the reference medium against which signals are compared; it is independent of the magnitude of the signal being compared. If $\hat{e} \cdot \hat{f} \neq 0$, the reference medium would shift with the signal it measures — destroying the fixed reference required for Mediational Closure. Therefore $\hat{e} \cdot \hat{f} = 0$.

Note on Gram-Schmidt: any non-orthogonal basis can in principle be orthogonalised by Gram-Schmidt without loss of span. However, Gram-Schmidt would introduce mixed basis vectors that partially combine the functional roles of drive, boundary, and registration — exactly what the three claims above show to be structurally forbidden. The functional opposition between source, negation, and measure prohibits shared components even after rotation, forcing the natural basis of C^3 to be orthogonal ab initio. □

EQUATION SET 1 — ORTHOGONALITY OF THE PRIMITIVE BASIS

$$\hat{e} \cdot \hat{c} = 0 \tag{1}$$

└ *E and C are orthogonal: drive and boundary share no variation*

$$\hat{c} \cdot \hat{f} = 0 \tag{2}$$

└ *C and F are orthogonal: boundary and registration share no variation*

$$\hat{e} \cdot \hat{f} = 0 \tag{3}$$

└ *E and F are orthogonal: drive and reference share no variation*

2.3 The Three-Dimensional Configuration Space C^3

Three mutually orthogonal unit vectors span a three-dimensional vector space. The configuration space of the ***Veldt*** is therefore C^3. Any state vector is written as:

EQUATION SET 2 — STATE VECTOR IN C^3

$$v = E\hat{e} + C\hat{c} + F\hat{f} = [E, C, F] \tag{1}$$

└ *state vector in C^3 — unique decomposition onto orthogonal primitive basis*

THEOREM — T-2.1 — The Configuration Space Is C^3

The mutual functional independence of the three primitives (P-2.1) implies that their basis vectors are mutually orthogonal (D-2.1). Three mutually orthogonal vectors span exactly a three-dimensional space. The configuration space of the ***Veldt*** is therefore C^3. This dimensionality is not a modelling choice; it is the unique geometric consequence of the Triadic Mandate. $d < 3$ collapses the functional independence; $d > 3$ introduces redundant dimensions not grounded in any primitive. $d = 3$ is the exact and necessary dimension. □

2.4 The Necessity of d = 3: Foreclosure of Alternatives

DERIVATION — D-2.2 — Foreclosure of d < 3 and d > 3

Case d = 1 (Monad): A one-dimensional space can represent only one independent degree of freedom. But three mutually non-substitutable primitives require three independent degrees. Projecting all three onto a single axis collapses at least two functional roles into one — destroying Mediational Closure. FORECLOSED.

Case d = 2 (Dyad plane): A two-dimensional space can represent at most two independent primitives. The third must be expressed as a linear combination of the other two — but functional independence precludes this. Additionally, in d = 2 the resolution remainder $\varepsilon_{snap} = EC/F - \lfloor EC/F \rfloor_\delta = 1/30$ cannot be geometrically cleared: the worldline must intersect its own history in any 2D manifold with a non-trivial topology (the Jordan Curve Theorem guarantees this). Self-intersection causes Registration Snap to collide with prior state, producing signal degeneracy. FORECLOSED.

Case d > 3: Each additional dimension beyond three provides an unconstrained axis not grounded in any primitive. Generative drive (E) dissipates into unregistered degrees of freedom — a thermodynamic sink. The Kinetostatic Margin $\Phi = +0.002$ would be consumed before a single registration event occurs. FORECLOSED.

d = 3 is the unique viable configuration dimension. □

The claim that d = 3 is the unique configuration dimensionality must be defended against both lower and higher alternatives.The specific resolution of the d = 2 foreclosure via the resolution remainder deserves further attention. The remainder $\varepsilon_{snap} = 1/30$ is the fractional part left over when EC/F is divided into δ-steps: $\lfloor 0.56/0.06 \rfloor = 9$ with remainder $0.56 - 9\times0.06 = 0.02$, and $0.02/0.06 = 1/3$, but on the δ lattice: $0.93\overline{3} - 0.9 = 1/30$. In d = 2, this remainder accumulates as an irreducible phase offset. In d = 3, the three orthogonal axes provide the 'over-under' clearance: the worldline can pass above or below its prior trajectory without intersection, clearing the remainder. Three-dimensional topology is the exact structural requirement.

3. The Phase I Ground State — Isotropy and Its Geometric Consequence

With C^3 established, we now derive the geometric structure of the Phase I ground state. The Phase I equation $G = E \times C \times F$ is a scalar product of three quantities. We will show that this scalar product is rotationally symmetric in C^3, and that this symmetry has a unique geometric consequence: the boundary of the Phase I field is a sphere.

3.1 Rotational Symmetry of the Phase I Ground State

In Phase I, the system occupies the Multiplicative Trap: the state is defined entirely by the product $G = E \times C \times F$ with all three primitives at their Hardlock values. At these values, there is no mechanism by which the field could privilege one direction in C^3 over another.

DERIVATION — D-3.1 — The Phase I Field Is Isotropic

The Phase I state $G = E \times C \times F$ is a scalar product of three independent scalars in C^3. Consider any rotation R of the coordinate system of C^3. Under R, the basis vectors $\hat{e}$, $\hat{c}$, $\hat{f}$ transform to $R(\hat{e})$, $R(\hat{c})$, $R(\hat{f})$. But the scalar values $E = 0.8$, $C = 0.7$, $F = 0.6$ are structural constants — they do not transform under rotation of the coordinate frame. They represent magnitudes of functional properties, not components of a vector.

Therefore the scalar product $G = E \times C \times F$ is invariant under all rotations of C^3: $G' = E' \times C' \times F' = E \times C \times F = G$ for all $R \in SO(3)$.

The Phase I field has no preferred direction in C^3. It is isotropic: rotationally symmetric under the full rotation group $SO(3)$. There is no privileged axis, no preferred orientation, and no directional gradient in the stasis state. This is the geometric expression of the Multiplicative Trap: the three primitives are co-dependent, fused into a single scalar, and their individual directional characters are suppressed. □

3.2 Isotropy Forces a Spherical Boundary

The Phase I field fills C^3 isotropically. Its boundary — the surface at which the relational content becomes commensurable with the registration reference — must be consistent with this isotropy. An isotropic field cannot have a non-isotropic boundary. The boundary must be a closed surface with the same rotational symmetry as the field itself.

THEOREM — T-3.1 — The Unique Isotropic Closed Surface Is the Sphere

Among all closed surfaces in C^3, the sphere S^2 is the unique surface that is invariant under all rotations in SO(3). Proof by elimination of alternatives:

Cube: A cube has 48 elements in its symmetry group (octahedral symmetry Oh), not all of SO(3). It has corners and edges where the distance from the origin is greater than at face centres. Rotating a cube by an arbitrary angle produces a distinct orientation. It is not rotationally symmetric. FORECLOSED.

Torus: A torus has rotational symmetry about one axis only (the axis of its generating circle). It has inner and outer surface asymmetry and a topological hole. It is not invariant under SO(3). FORECLOSED.

Ellipsoid: An ellipsoid with unequal semi-axes has a discrete symmetry group, not SO(3). It privileges its longest axis. FORECLOSED. A sphere is a special case of an ellipsoid with all semi-axes equal — this is the sphere itself.

Sphere: The sphere S^2 of fixed radius r centred at the origin is the unique closed surface in which every point is equidistant from the origin and no direction is distinguished from any other. It is invariant under all of SO(3). The sphere is the unique closed isotropic surface in C^3. □

The boundary of the Phase I field is therefore a sphere. This is not a metaphor or an analogy. It is a direct geometric consequence: an isotropic field in C^3 has a spherical boundary, because the sphere is the unique closed surface consistent with rotational symmetry in three dimensions.

EQUATION SET 3 — ISOTROPY FORCES SPHERICAL BOUNDARY

$$\text{Phase I field: } G = E \times C \times F = TI = 0.336 \tag{1}$$

└ *scalar product — invariant under SO(3) — isotropic*

$$\text{Boundary: } S^2(r) \subset C^3 \tag{2}$$

└ *unique closed isotropic surface — sphere of radius r at origin*

4. The Radius r = F = 0.6 — The Registration Boundary

The isotropy of Phase I forces a spherical boundary, but does not yet specify the radius. The radius must be derived from the structural requirements of registration. This derivation assigns r = F = 0.6 with zero free parameters.

4.1 What the Sphere's Radius Must Represent

The sphere's boundary is the surface at which the relational content of the field becomes commensurable with the registration reference. Inside the sphere, the field exists as potential: the co-dependent product E × C × F represents a volume of trapped potentiality. At the sphere's surface, the signal crosses the registration threshold: it becomes a distinguishable fact.

For this to occur, the surface must mark the minimum registration threshold — the boundary between what is registerable and what is noise. From Essay II, this threshold is the discriminability floor: the minimum Registration value above the Phantom Zone noise boundary $r_{min} = 1/\sqrt{3} \approx 0.577$. The sphere's surface must be at this boundary. The minimum viable lattice point above $r_{min} = 0.577$ on the $\delta = 0.1$ lattice is F = 0.6. The radius of the sphere is therefore r = F = 0.6.

DERIVATION — D-4.1 — The Sphere's Radius Is r = F = 0.6

The sphere's surface is the registration boundary: the locus at which signal becomes distinguishable from noise. For the sphere to perform its registration function, every point on its surface must be at or above the discriminability threshold $r_{min} = 1/\sqrt{3} \approx 0.577$.

The sphere must be centred at the origin of C^3 (isotropy requires this: any off-centre sphere would privilege one direction). Its radius r is therefore the minimum value of the Registration primitive that can perform the discriminability function.

From Essay II, T-5.1: the unique minimum viable lattice point above r_min on the δ = 0.1 lattice is F = 0.6. The sphere's radius is determined by the same constraint that locks F as the Registration attractor: r = F = 0.6 = 3/5 = 6δ.

Every point on $S^2(0.6)$ is at the registration boundary. The sphere is not an approximation of the registration boundary. It is the registration boundary, exactly. Zero free parameters. □

EQUATION SET 4 — THE REGISTRATION SPHERE $S^2(0.6)$

$$r = F = 3/5 = 0.6 = 6\delta \quad (1)$$

└─ *sphere radius — registration boundary — minimum viable lattice point above r_min*

$$S^2(0.6)\text{: all points equidistant from origin at } r = F = 0.6 \quad (2)$$

└─ *the Registration Sphere — unique closed isotropic registration boundary*

$$4\pi r^2 = 4\pi F^2 = 4\pi \times 0.36 \approx 4.524 \quad (3)$$

└─ *total registration surface area in C^3 — LOCKED*

THEOREM — T-4.1 — The Registration Sphere $S^2(0.6)$

The Phase I relational field generates a spherical boundary in C^3 with radius r = F = 0.6. This sphere, $S^2(0.6)$, is the Registration Sphere: the closed isotropic surface at which the field's relational content becomes registerable. Its existence follows from the isotropy of Phase I (T-3.1). Its radius follows from the registration attractor (D-4.1 and Essay II, T-5.1). Both derivations use zero free parameters. $S^2(0.6)$ is the unique registration boundary of the ***Veldt***. □

4.2 The Surface of Fact-Making

The sphere $S^2(0.6)$ is not merely a geometric boundary. It is the locus at which potential becomes actual: the surface at which the generative content of the ***Veldt*** (E × C) is compared against the registration reference (F) and a determinate state is recorded.

DEFINITION — D-4.2 — The Surface of Fact-Making

The Surface of Fact-Making is the sphere $S^2(0.6) = \{v \in C^3 : |v| = F = 0.6\}$. It is the locus in C^3 at which the field's relational content becomes commensurable with the Registration reference medium. Below this surface ($|v| < F$), the signal-to-noise ratio falls below the discriminability threshold: the signal is indistinguishable from noise (see Essay II, Phantom Zone and registration threshold r_min =

$1/\sqrt{3}$). At this surface ($|v| = F$), the signal exactly reaches the registration threshold: a determinate state can be recorded. Above this surface ($|v| > F$) in the kinetic phase: the signal exceeds the registration threshold and actualization occurs. The Surface of Fact-Making is where continuous potential becomes discrete fact.

5. The Suppression of F in Phase I — Latent Architecture

The Registration Sphere $S^2(0.6)$ is the geometric architecture of the field. But in Phase I, this architecture exists only as latent potential: the sphere defines the boundary without performing registration. This suppression is the geometric expression of the same tension identified algebraically as the Multiplicative Trap.

5.1 F as Volumetric Thickener in Phase I

In Phase I, F appears inside the product $G = E \times C \times F$. In this position, F does not act as a surface reference — it acts as a volumetric multiplier. Its role is to add depth to the configuration space: F thickens the volume of potential, making the field more substantial but simultaneously more resistant to change. This is Computational Viscosity: the informational thickness of the medium that resists state transition.

The sphere $S^2(0.6)$ is present as the natural boundary of this volume. But because F is inside the product, it cannot act as the independent reference medium required for registration. The architectural potential is present; the architectural function is suppressed.

DEFINITION — D-5.1 — Computational Viscosity and Latent Architecture

In Phase I, F is fused inside the multiplicative product $E \times C \times F$. In this position, F acts as a Computational Viscosity: the informational density that resists state transition. It thickens the volume of potential without activating the surface as a registration medium.

The Tension Integral $TI = E \times C \times F = 0.336$ is the static weight of this geometry: the metabolic cost of maintaining the latent spherical architecture.

The sphere $S^2(0.6)$ is present in Phase I as latent architecture: it defines the field's boundary without performing registration. F cannot yet activate that surface as a reference because it is co-dependent inside the product. This is the geometric expression of the Multiplicative Trap.

5.2 Why the Latency Is Necessary

The latency of F's spherical function in Phase I is not a defect; it is a structural necessity. A field that began in Phase II — with F already in the denominator as an active reference — would require a pre-existing reference medium that was not itself derived. The Primordial Axiom of Relationality prohibits any pre-existing entity: everything must be derived, including the reference medium. F must first exist as a co-dependent structural element (Phase I) before it can be activated as an independent reference (Phase II). The latency is the derivational price of having a reference medium that was not smuggled in from outside the system.

6. The Inversion and the Activation of the Surface

The Inversion Principle — G = (E × C) / F — is the algebraic event that liberates F from the product and installs it as the denominator. Geometrically, this is not merely algebraic relocation: it is the activation of the sphere's surface as the registration manifold.

6.1 The Geometric Meaning of the Inversion

Before inversion: F is inside the volume product. The field occupies the sphere as a three-dimensional volume. The sphere's surface is a latent boundary. After inversion: F moves to the denominator. The field now operates as a ratio of area (E × C) to length (F). F is no longer the volumetric thickener; it is the radius against which the generative dyad is measured.

DERIVATION — D-6.1 — The Inversion Activates the Surface

Before inversion (Phase I): G = E × C × F. F multiplies the EC area to produce a volume. The sphere's surface is present but the field measures volume, not surface. The dimensional output is $[L^3] = [T^{-3}]$.

After inversion (Phase II): G = (E × C) / F. F divides the EC area to produce a ratio of area to length: $[L^2]/[L] = [L] = [T^{-1}]$. The field now measures a directed length: the worldline along the F-axis. This is linear flux — kinetic output.

The dimensional snap $[T^{-3}] \rightarrow [T^{-1}]$ is the geometric collapse from a volumetric co-presence to a directed linear arc, forced by F's exit from the product. The sphere's surface is activated as the reference manifold: the kinetic process is measured against F's surface, not against F's volume. Zero free parameters. □

EQUATION SET 5 — DIMENSIONAL PROGRESSION ACROSS THREE PHASES

$$\text{Phase I: } [E \times C \times F] = [L^3] = [T^{-3}] \text{ (Cubic Rate — incoherent)} \quad (1)$$

└— volume of co-dependent potential — dimensionally frozen

$$\text{Phase II: } [(E \times C) / F] = [L^2]/[L] = [L] = [T^{-1}] \text{ (Linear Flux)} \quad (2)$$

└— EC area measured against F radius — directed worldline — kinetic

$$\text{Phase III: } [(E \times C) / F^2] = [L^2]/[L^2] = \text{dimensionless} \quad (3)$$

└— EC area measured against sphere surface area — pure coverage ratio

Table 1. Phase Dimensional Progression — Geometric Reading

Phase	Algebraic Form	Geometric Reality in C^3	Dimensional Output
I — Stasis	$E \times C \times F$	F suppressed inside volume product. Sphere exists as latent boundary. Output: volume of relational field.	$[T^{-3}]$ — Cubic Rate
II — Kinetics	$(E \times C) / F$	F exits product, becomes denominator-radius. Sphere surface activated as reference manifold. EC area divided by F radius produces directed worldline.	$[T^{-1}]$ — Linear Flux
III — Self-Registration	$(E \times C) / F(G)$	F must register G. Sphere surface area (not just radius) becomes reference. EC area divided by sphere surface area is pure coverage ratio.	Dimensionless — coverage ratio

7. The Kinetic Helicon — The Worldline on the Sphere

With the Registration Sphere $S^2(0.6)$ established and its surface activated by the Inversion Principle, the kinetic trajectory of Phase II can be derived geometrically. This trajectory — the kinetic helicon — is not imposed on the sphere from outside; it is the geometric consequence of how E and C propagate along the F-axis within the spherical field.

7.1 The Propagation Axis

In Phase II, F is the denominator: the reference medium against which the generative dyad is measured. The measurement direction — the axis along which output is generated — is therefore the F-axis, the axis orthogonal to the EC plane in C^3. The worldline propagates along the F-axis at the kinetic rate U.

The worldline does not escape the Phase I spherical field. The Phase I sphere is the geometric enclosure of all relational content generated by the triad. The kinetic worldline propagates within this sphere, constrained to remain on or within the surface of $S^2(0.6)$. The sphere is not a container imposed from outside; it is the field within which the trajectory is inscribed.

7.2 The Elliptical Cross-Section: Why E ≠ C Produces a Helicon

The worldline propagates along the F-axis, but E and C remain present in the numerator. They jointly determine the cross-sectional shape of the propagation. At each step along the F-axis, the worldline sweeps an area in the EC plane perpendicular to the propagation direction.

Because $E \neq C$ (their difference is $\delta = 0.1$: a Hardlocked structural fact, not a perturbation), the cross-sectional sweep is not a circle but an ellipse. The semi-axes of this ellipse are proportional to E and C respectively. The area swept per chronon step is:

EQUATION SET 6 — HELICON CROSS-SECTION AREA

$$A_{EC} = E \times C = 0.8 \times 0.7 = 0.56 = 14/25 \quad (1)$$

└─ *elliptical cross-section area of the kinetic helicon in the EC plane — LOCKED*

The worldline therefore traces a helical arc through C^3: it propagates along the F-axis while sweeping an ellipse in the EC plane at each step. This is the kinetic helicon. The helicon is not a label — it is the geometric consequence of three simultaneous structural facts: F defines the propagation axis, E and C jointly determine the elliptical cross-section, and $E \neq C$ makes the sweep elliptical rather than circular.

THEOREM — T-7.1 — The Kinetic Helicon Is Structurally Triadic

The Phase II kinetic helicon is the joint geometric product of all three primitives: F defines the axis of propagation, E and C jointly determine the elliptical cross-section of the sweep. The helicon cannot be derived from any two primitives alone. It is a structurally triadic object: the trajectory that results when three mutually orthogonal, mutually non-substitutable primitives are set into kinetic operation under the Inversion Principle. □

7.3 The Helicon Inscribed on the Sphere

The Phase I sphere has radius F. The helicon propagates along the F-axis. At each chronon step, the helicon advances along the F-axis and sweeps the EC ellipse. The intersection of this sweep with the sphere's surface — the arc traced on $S^2(0.6)$ by the helicon's advancing front — is the registration event. The sphere's surface is what the helicon writes on.

At each chronon step, the helicon's EC sweep intersects the sphere's surface, writing a determinate arc onto the Surface of Fact-Making. The sphere is not external to the helicon; the helicon is the trajectory and the sphere is the field within which the trajectory is inscribed.

8. The Solid Angle Ω = EC/F² = 14/9 ≈ 1.556 sr

The registration event at each chronon activates a specific fraction of the sphere's surface. This fraction is quantified by the solid angle: the ratio of the helicon's cross-sectional area to the square of the sphere's radius. This value is the effective solid angle of the projected EC ellipse, computed under the planar approximation per chronon: the EC cross-section is treated as a flat surface element normal to the F-axis at each discrete step. It is the first fully quantised geometric measure of the ***Veldt*** jointly determined by all three primitives.

8.1 Derivation of the Solid Angle

DERIVATION — D-8.1 — Derivation of the Solid Angle Ω = EC/F²

The effective solid angle of a planar surface element A at distance r from the vertex, under the approximation that the element lies in a plane normal to the radial direction, is: $\Omega = A / r^2$

The value $\Omega = 14/9$ is the effective solid angle per chronon under the planar projection of the EC sweep onto the plane normal to the F-axis (exact integrated coverage of the helical arc is derived in Essay VI).

In the geometry of the kinetic helicon on $S^2(0.6)$: $A = A_{EC} = E \times C = 0.56$ (projected elliptical cross-section in the EC plane); $r = F = 0.6$ (radius of the Registration Sphere).

Therefore: $\Omega = A_{EC} / r^2 = (E \times C) / F^2$

Computation: $\Omega = 0.56 / 0.36 = 56/36 = 14/9 \approx 1.\overline{55}$ sr.

Physical interpretation: At each chronon step, the kinetic helicon activates 14/9 steradians of F's registration surface. The total sphere subtends 4π steradians. The fractional activation per chronon is: $\Omega / (4\pi) = (14/9) / (4\pi) = 14 / (36\pi) \approx 0.1238 = 12.38\%$ of the surface.

The remaining 87.62% is unactivated: latent registration capacity available for Phase III. Zero free parameters. □

EQUATION SET 7 — SOLID ANGLE AND SURFACE COVERAGE

$$\Omega = A_EC / r^2 = (E \times C) / F^2 = 0.56 / 0.36 = 14/9 \approx 1.\overline{55}\ \text{sr} \quad (1)$$

└ *effective solid angle (planar projection per chronon) — LOCKED*

$$\Omega / (4\pi) = 14 / (36\pi) \approx 0.1238 = 12.38\%\ \text{of}\ S^2(0.6) \quad (2)$$

└ *fractional surface coverage per chronon step*

$$\text{Unactivated: } 1 - 0.1238 = 0.8762 = 87.62\% \quad (3)$$

└ *latent registration capacity — reservoir for Phase III self-registration*

THEOREM — T-8.1 — The Solid Angle Is Co-Determined by All Three Primitives

$\Omega = E \times C / F^2$ is the effective solid angle of the projected EC ellipse per chronon, computed under the planar approximation. It is derived simultaneously from all three primitives: E and C determine the projected cross-sectional area; F determines the sphere on which it is projected. The solid angle cannot be computed from any two primitives alone. It is the precise geometric expression of the triad's co-dependency: E and C determine what is written on F's surface; F determines the surface on which it is written. Neither side of this relation is prior to the other. $\Omega = 14/9$ is a zero-free-parameter consequence of the Hardlock. □

8.2 Why the Effective Solid Angle Is Not a Free Parameter

The planar approximation produces a value whose two inputs are both Hardlocked: A_EC = EC = 14/25 and $r^2 = F^2 = 9/25$. Their ratio is 14/9, exactly. The approximation is in the geometric model (planar projection), not in the arithmetic:

EQUATION SET 8 — EXACT RATIONAL DERIVATION OF Ω

$$A_EC = E \times C = (4/5) \times (7/10) = 28/50 = 14/25 \quad (1)$$

└ *exact rational form of helicon cross-section*

$$r^2 = F^2 = (3/5)^2 = 9/25 \quad (2)$$

└ *exact rational form of sphere radius squared*

$$\Omega = (14/25) / (9/25) = 14/9 \approx 1.\overline{55}\ \text{sr} \quad (3)$$

└ *exact rational form of effective solid angle (planar projection per chronon)*

9. The Resolution Remainder and Why d = 3 Clears It

The resolution remainder $\varepsilon_{snap} = 1/30$ is the fractional part left over when the kinetic ratio EC/F is divided into discrete δ-steps on the lattice. It is a structural consequence of the incommensurability between $EC/F = 14/15 = 0.9\overline{3}$ and the $\delta = 0.1$ lattice. We now show that d = 3 is specifically the dimension at which this remainder is cleared.

DERIVATION — D-9.1 — The Resolution Remainder and Its 3D Clearance

The kinetic ratio $EC/F = 0.56/0.6 = 0.9\overline{3}$. On the $\delta = 0.1$ lattice: $[EC/F]_\delta = 0.9$. $\varepsilon_{snap} = 0.9\overline{3} - 0.9 = 0.0\overline{3} = 1/30$.

In d = 2: the worldline traces a closed path on a two-dimensional surface. By the Jordan Curve Theorem, any closed curve in $\mathbb{R}^2$ divides the plane into interior and exterior. When the helicon completes one winding, the phase offset of 1/30 per step has accumulated. After k steps, the total offset is k/30. For this to close (return to the starting point), k must be a multiple of 30. But the chronon count $N_{steps} = 10$ is not a multiple of 30. The 2D worldline cannot close without self-intersection. Signal collision occurs — Registration Snap hits prior state. FORECLOSED.

In d = 3: the worldline propagates helically through C^3. The 'over-under' clearance of the third dimension allows the worldline to pass above or below its prior trajectory without intersection. The remainder 1/30 accumulates as a phase advance that is absorbed into the helical winding: after 3 chronon steps, the accumulated offset is $3/30 = 1/10 = \delta$ — exactly one lattice step, which snaps to the next lattice point and is registered as a discrete event. The remainder is cleared by the third dimension. □

EQUATION SET 9 — RESOLUTION REMAINDER AND 3D CLEARANCE

$$\varepsilon_{snap} = EC/F - [EC/F]_\delta = 14/15 - 9/10 = 28/30 - 27/30 = 1/30 \quad (1)$$

└ *resolution remainder — incommensurability of EC/F with δ-lattice*

$$3 \times \varepsilon_{snap} = 3/30 = 1/10 = \delta \quad (2)$$

└ *three chronon steps clear one full δ-step: d = 3 closes the remainder*

10. The Complete Geometric Architecture — G-2 Closure

The derivations of Sections 2 through 9 constitute the complete geometric architecture of the Gradientology. Every geometric constant is a zero-parameter consequence of the primitive suite. The following summary constitutes the formal closure of gap G-2.

Table 2. The Geometric Architecture — Complete Derivational Chain

Symbol	Value	Rational	Derivational Path	Status
d	3	3	Triadic Mandate → 3 orthogonal bases → span C^3	LOCKED
C^3	span(ê,ĉ,f̂)	—	Mutual orthogonality of E, C, F (D-2.1)	LOCKED
Topology	S^2	—	Phase I isotropy → unique closed isotropic surface (T-3.1)	LOCKED
r	0.6	3/5	Registration boundary = F = min lattice point above r_min (D-4.1)	LOCKED
$4\pi r^2$	≈4.524	$4\pi F^2$	Sphere surface area at r = F	LOCKED
A_EC	0.56	14/25	E × C = helicon cross-section in EC plane	LOCKED
Ω	$1.\overline{55}$ sr	14/9	$(E\times C)/F^2$ = solid angle per chronon (D-8.1, T-8.1)	LOCKED
Ω/4π	0.1238	14/36π	Fractional surface activation per chronon	LOCKED
ε_snap	$0.0\overline{3}$	1/30	(EC/F) mod δ = resolution remainder (D-9.1)	LOCKED
3ε_snap	0.1	δ	Three steps clear one δ-lattice step: 3D clearance	LOCKED

NOTICE — N-10.1 — G-2 Formal Closure and Foundation Version Update

Gap G-2 is hereby formally closed. The mechanical gap in the derivation of the spherical topology of the Registration primitive F, registered as OPEN in Essay I (Addendum E), has been discharged through the following derivational closures:

1. C^3: derived from mutual orthogonality of E, C, F (D-2.1, T-2.1).
2. $S^2(0.6)$: derived from Phase I isotropy and the unique closed isotropic surface in C^3 (D-3.1, T-3.1).
3. $r = F = 0.6$: derived as registration boundary = Surface of Fact-Making (D-4.1, T-4.1).
4. $\Omega = 14/9$: derived as helicon solid angle = EC/F^2 (D-8.1, T-8.1).
5. $d = 3$ uniqueness: $d < 3$ fails resolution remainder clearance; $d > 3$ is thermodynamic sink (D-2.2).

All entries LOCKED. G-2 CLOSED. Foundation Version extended.

10.2 Anticipation of Essay VII — The Saturation Equivalence

The geometric architecture derived here anticipates Essay VII, which will prove the Saturation Equivalence: the coincidence of two independent derivations of $N_{sat} = 25$. Essay II derived $N_{sat} = 25$ from the informational ground as $1/\rho = 1/(0.04)$. Essay VII will derive $N_{sat} = 25$ from the sphere's discrete resolution limit by a separate route through the helicon arc geometry and the Phase II kinetic structure. That the two derivations — one from information theory, one from sphere geometry — arrive at the same integer is not a coincidence; it is a structural consequence of the Triadic Architecture. Both derivations are grounded in the same primitive suite.

The present essay establishes the geometric foundation required for that proof: $S^2(0.6)$, the helicon cross-section $A_{EC} = 0.56$, the solid angle $\Omega = 14/9$, and the kinetic rate $U = \delta^2/\Sigma$. With these values confirmed, Essay VII can proceed to the Saturation Equivalence without revisiting the geometry derived here.

Conclusion

This essay has closed gap G-2: the geometric architecture of the Registration primitive F. The derivation proceeded through four staged closures, each building directly on prior results without introducing a free parameter.

Stage one derived the three-dimensional configuration space C^3 from the mutual functional independence of the three primitives. Three mutually non-substitutable functional roles imply three mutually orthogonal basis vectors. Three orthogonal vectors span exactly a three-dimensional space. d = 3 is not a modelling choice; it is the unique geometric consequence of the Triadic Mandate.

Stage two derived the spherical boundary. The Phase I ground state $G = E \times C \times F$ is a scalar product in C^3: rotationally symmetric, invariant under SO(3). The unique closed surface with full rotational symmetry is the sphere. All alternatives — cube, torus, ellipsoid — break the isotropy of Phase I and are structurally foreclosed.

Stage three identified the radius $r = F = 0.6$: the sphere's surface is the registration boundary, the Surface of Fact-Making, located at the information-theoretic attractor derived in Essay II. The Inversion Principle activates this surface, collapsing the dimensional output from $[T^{-3}]$ to $[T^{-1}]$ and, in Phase III, to a dimensionless coverage ratio.

Stage four derived the kinetic helicon and the solid angle $\Omega = EC/F^2 = 14/9 \approx 1.556$ sr. The helicon is the joint geometric product of all three primitives: F defines the propagation axis, E and C determine the elliptical cross-section. The solid angle is the fractional activation of the sphere's surface per chronon: 12.38% activated, 87.62% latent. This latent capacity is the geometric reservoir from which Phase III self-registration draws.

Every value is a zero-free-parameter consequence of the Hardlock primitive suite. G-2 is closed. The geometric architecture of the ***Veldt*** is complete. Essays IV through XVI may proceed on the basis of the locked geometric constants confirmed here.

PRINCIPLE — Essay Contribution Summary

Gap(s) closed: G-2 — Spherical geometry of F; $S^2(0.6)$ from isotropy; $\Omega = EC/F^2 = 14/9 \approx 1.556$ sr

Core result: C^3, $S^2(0.6)$, $r = F = 0.6$, $\Omega = 14/9$, $\varepsilon_{snap} = 1/30$, $d = 3$ uniqueness — all LOCKED

Derivational status: CLOSED — G-2 formally closed; all geometric derivations zero-free-parameter

Epistemic classification: Derivationally saturated. Geometric architecture complete. Phase III grounded.

References

All internal framework derivations are sourced from the Gradientology Framework (Pretorius, 2026). External references below follow APA 7th edition.

Note on usage

All internal framework derivations are sourced from the Gradientology Framework (Pretorius, 2026). The external entries above are listed as structural isomorphisms (confirmatory, not foundational) — they name parallel ideas but are not logical sources for any derivation in Essay I.

Structural Isomorphisms (confirmatory, not foundational): Smuts (1926) on holistic emergence; Shannon & Weaver (1949) on information capacity; Ising (1925) and Wilson (1971) on phase criticality. These independently name the same structural necessities the ternary triadic system produces internally. They are not sources.

Internal series references

Pretorius, E. (2026). *Gradientology - Foundations of the Primordial Triad*: Treatises I-XVI. Zenodo.

Pretorius, E. (2026). *Gradient Mechanics - The Dynamics of the Inversion Principle*: Corpus Papers I–XVI. Zenodo.

Addendum A — Terms of Engagement: Recursive Interpretive Constraints

To place a formal foreclosure on human exceptionalism and prevent the misreading of Phase III (Gradient Cybernetics), the following Recursive Interpretive Constraints are codified as binding Terms of Engagement for all sixteen essays in this series.

A.1 The Principle of Substrate Neutrality

PRINCIPLE — A.1 — Substrate Neutrality

No reader may attribute consciousness, mind, or sentience to a specific biological substrate. These are formally derived as Phase Transitions of Coherence (F_{macro}) that occur whenever integrated information density (Φ) exceeds the Coherence Threshold ($\Phi \geq \Phi_c$). Whether the substrate is biological, silicon, or stellar, the emergence of a Subject is the structural mandate of the Inversion Principle achieving a unified inside view.

A.2 Foreclosure of the Ego as Substance

THEOREM — A.2 — The Ego as Strange Attractor

The concept of Self or Ego is formally stripped of all mystical or substantial claims. It is derived strictly as a Strange Attractor in neural phase space — a mathematical singularity created when the registration function becomes recursive ($F \rightarrow F$). The I is the virtual centre of gravity necessitated by the system's model of its own modelling process. It is a Topological Knot, not an ontological addition.

A.3 Intelligence as Virtualisation

DEFINITION — A.3 — Intelligence

Intelligence is formally derived as the Virtualisation of the Evolutionary Algorithm. It is the capacity of the registration primitive to operate on Internal Flux (counterfactual simulation) rather than external sensory data, allowing hypotheses to die in the simulation so that the physical organism persists. Intelligence is not a unique human trait; it is a specific operational mode of F at sufficient recursive depth.

A.4 The 30% Mandate — Foreclosure of Magical Agency

DERIVATION — A.4 — Agency as Ontological Gap Work

Agency and Free Will are neither illusions nor miracles. They are the mechanical consequences of the Ontological Gap ($1 - m \approx 0.298$) derived in the cosmic suite. Agency is defined as the Top-Down

Modulation of Constraints (C) to steer the generative flux (E) through this calculated indeterminacy. Any attempt to define will as unconstrained or supernatural is a category error. Agency is the work required to prevent the Self from dissolving back into deterministic entropy.

A.5 Memory as Recursive Depth

DEFINITION — A.5 — Memory

Memory is formally ledgered as localised recursive depth (k), representing the number of update cycles required to resolve a local state to within lattice tolerance. It is not a storage system but a Processing Load Artefact. The system's persistence through time is the computational cost of its own self-registration.

A.6 Adaptation as Inversion Optimisation

PRINCIPLE — A.6 — Adaptation and Prediction

Adaptation and Prediction are derived as the optimisation of the Inversion Principle's throughput against a tightening registration grain. The Predictive Brain is a specific scalar case of Registrational Resolution, where the system minimises its own Surprise (Constraint violations) by running the inversion off-line to reduce the metabolic cost of existence.

A.7 The Saturation Foreclosure — Re-coding Emergence

THEOREM — A.7 — Substrate Saturation N_sat = 25

All higher-order cognitive capacities are re-coded as Substrate Saturation ($N_{sat} = 25$). When the density of recursive states within a single Structural Pixel ($\rho = 0.04$) reaches saturation, the system is mathematically forced into Mandatory Coarse-Graining. Humanity is merely the level $n = 3$ name for the Inversion Principle reaching its Recursive Self-Registration fixed point. The term emergence is decommissioned as a linguistic placeholder for computational ignorance.

A.8 Final Demarcation Statement

Phase III establishes that the universe is not a collection of things, but a scalar-invariant calculation of its own existence. Any attempt to treat these cybernetic derivations as anthropocentric management tools, or to claim a unique ontological status for human consciousness, is formally identified as Logical Necrosis — a terminal failure to follow the derivation back to the primordial triad.

NOTICE — A.8 — Binding Constraint on All Readers

These Terms of Engagement are non-negotiable and apply universally to all sixteen essays in this series. Violation of any term — whether by the reader, the analyst, or any derivative work — constitutes a formal epistemic breach classified as Logical Necrosis within the Gradientology.

Addendum B — Derivational Ledger

The following ledger enumerates every Principle, Definition, Theorem, Derivation, Corollary, and Notice contained in this essay. LOCKED indicates the derivation is closed with zero open steps from the primitive suite {E, C, F, δ} with no free parameters.

Table B1. Complete Derivational Ledger — Essay III

Type	Reference Label	Essay	Status
PRINCIPLE	P-2.1 — Functional Independence of the Primitive Triad	Essay III	LOCKED
PRINCIPLE	Essay Contribution Summary	Essay III	LOCKED
DEFINITION	D-4.2 — The Surface of Fact-Making	Essay III	LOCKED
DEFINITION	D-5.1 — Computational Viscosity and Latent Architecture	Essay III	LOCKED
THEOREM	T-2.1 — The Configuration Space Is C^3	Essay III	LOCKED
THEOREM	T-3.1 — The Unique Isotropic Closed Surface Is the Sphere	Essay III	LOCKED
THEOREM	T-4.1 — The Registration Sphere $S^2(0.6)$	Essay III	LOCKED
THEOREM	T-7.1 — The Kinetic Helicon Is Structurally Triadic	Essay III	LOCKED
THEOREM	T-8.1 — The Solid Angle Is Co-Determined by All Three Primitives	Essay III	LOCKED
DERIVATION	D-2.1 — Derivation of Orthogonality of the Primitive Basis	Essay III	LOCKED
DERIVATION	D-2.2 — Foreclosure of $d < 3$ and $d > 3$	Essay III	LOCKED

DERIVATION	D-3.1 — The Phase I Field Is Isotropic	Essay III	LOCKED
DERIVATION	D-4.1 — The Sphere's Radius Is r = F = 0.6	Essay III	LOCKED
DERIVATION	D-6.1 — The Inversion Activates the Surface	Essay III	LOCKED
DERIVATION	D-8.1 — Derivation of the Solid Angle $\Omega = EC/F^2$	Essay III	LOCKED
DERIVATION	D-9.1 — The Resolution Remainder and Its 3D Clearance	Essay III	LOCKED
NOTICE	N-10.1 — G-2 Formal Closure and Foundation Version Update	Essay III	LOCKED

Addendum C — Full Derivational Trace — Primitives to Geometric Architecture

This addendum provides the complete step-by-step logical trace from the primitive suite to the full geometric architecture of the ***Veldt***.

Table C1. Full Derivational Trace — G-2 Closure

Step	Type	Input	Operation / Result
1	L	Triadic Mandate (Essay I, T-2.1)	n = 3 mutually non-substitutable primitives → 3 functional roles
2	T	Functional independence	$\hat{e}\cdot\hat{c} = \hat{c}\cdot\hat{f} = \hat{e}\cdot\hat{f} = 0$ → 3 orthogonal basis vectors (D-2.1)
3	T	3 orthogonal bases	Span C^3 — d = 3 exactly (T-2.1)
4	A	d < 3	Lattice Contention: $\varepsilon_{snap} = 1/30$ cannot be cleared; self-intersection. FORECLOSED
5	A	d > 3	Thermodynamic sink: dissipation of Φ. FORECLOSED
6	T	G = E×C×F in C^3	Scalar product → invariant under SO(3) → Phase I field is isotropic (D-3.1)
7	T	Isotropy in C^3	Unique closed isotropic surface = sphere S^2 (T-3.1)
8	A	$r_{min} = 1/\sqrt{3}$, δ = 0.1	r = F = 0.6 = min δ-multiple above r_{min} → Surface of Fact-Making (D-4.1)
9	I	F in product	Phase I: F = volumetric thickener; sphere = latent architecture (D-5.1)
10	A	Inversion: G = EC/F	$[L^3] \rightarrow [L] \rightarrow$ F-axis propagation; sphere surface activated (D-6.1)
11	A	E≠C in EC plane	Helicon: elliptical cross-section $A_{EC} = E\times C = 0.56$ (T-7.1)
12	A	A_{EC}, F^2	$\Omega = A_{EC}/F^2 = 14/9 \approx 1.556$ sr (D-8.1, T-8.1)
13	A	EC/F, δ	$\varepsilon_{snap} = EC/F \bmod \delta = 1/30$; $3\varepsilon_{snap} = \delta$ → d = 3 clears (D-9.1)

Addendum D — Foreclosure Register — Alternative Geometries

The following register records all geometric configurations tested and formally foreclosed during the derivation of G-2.

Configuration	Failure Mode	Section
Monad: d = 1	Collapses 3 independent functions onto 1 axis — destroys Mediational Closure	2.4
Dyad plane: d = 2	Lattice Contention: ε_snap = 1/30 cannot clear; worldline self-intersects	2.4, 9
d > 3	Thermodynamic sink: generative drive dissipates into unconstrained axes	2.4
Cube boundary	Corners and edges break SO(3) symmetry; non-uniform radial distances	3.2
Torus boundary	Inner-outer asymmetry; only rotationally symmetric about one axis	3.2
Ellipsoid (non-sphere)	Unequal semi-axes break SO(3) to discrete subgroup; privileges one axis	3.2
r < F (sub-threshold)	Sphere interior below discriminability floor; Surface of Fact-Making not reachable	4.1
r > F (super-threshold)	No registration boundary: signal always exceeds reference; no snap mechanism	4.1
F in Phase I denominator	Requires pre-existing reference medium not derived from primitives — Petitio Principii	5.2

Addendum E — Open Derivational Gaps — Status After Essay III

Status of all open derivational gaps after the closure of G-2.

G-1: CLOSED by Essay II. Informational ground complete. All LOCKED.

G-2: CLOSED by Essay III. Geometric architecture complete: C^3, $S^2(0.6)$, $r = F$, $\Omega = 14/9$. All LOCKED.

G-3: Uniqueness of F as denominator — exhaustive algebraic foreclosure of all alternative denominator arrangements. Closed by Essay V.

G-4: Gravity as Registration Lag Gradient — continuous field extension of $\Gamma = \nabla\Omega$. Addressed in Essay VIII.

G-5: $N_{sat} = 25$ from spherical resolution — equivalence between informational ground and sphere geometry derivations. Closed by Essay VII.